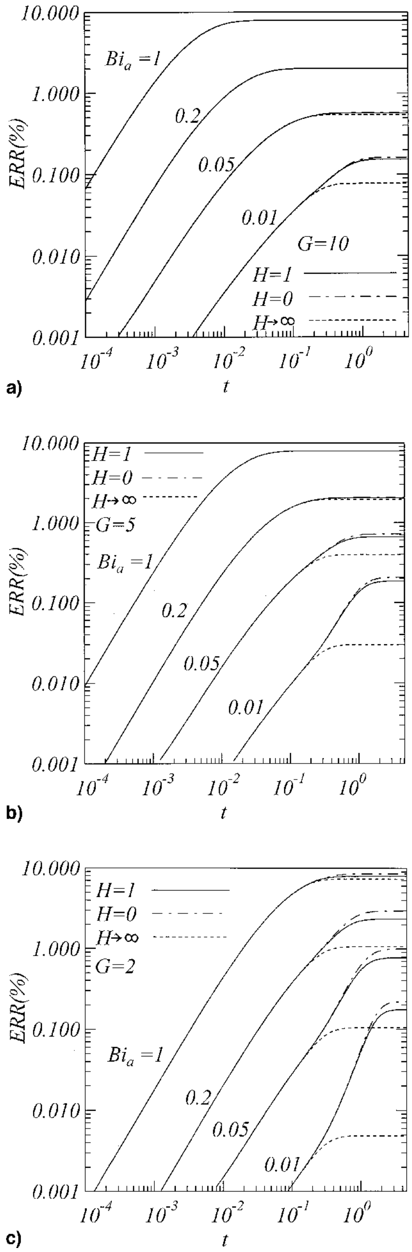

Fig. 1 Effects of Bi_a on relative error between one- and two-dimensional transient heat flow rate at the base. G = a) 10, b) 5, and c) 2.

results obtained in Eqs. (8) and (9) are in exact agreement with Tables III and II of Carslaw and Jaeger.[7] The convergence of the series' expressions for temperature distribution and heat flow is rather slow for small values of time ($t \leqq 0.01$). Therefore, it is required to determine the 250th root in Eq. (9). In each case, the summing process is terminated when the value of the last term computed is less than 0.000001.

Figures 1a–1c show that the relative errors of the base heat flow between one- and two-dimensional transient solutions are affected by H for different Bi_a at G = 10, 5, and 2. The relative errors increase with increasing Bi_a and t, and decrease with G at the fixed value of H. The steady-state condition is reached later as the values of G and Bi_a decrease. The relative error is insensitive to the variation in H for smaller times at the different Bi_a number. For fixed values of $Bi_a \leqq 0.2$, the effect of H on the relative error becomes more significant as G decreases at large times, but the effect of H on the relative error is insignificant when $Bi_a = 1.0$ and the different values of G. Inspection of Figs. 1a–1c reveals that the relative error is a minimum value as $H \to \infty$ and is a maximum value at $H = 0$ for the cases of $Bi_a = 0.01$ with various G values when steady state is reached. Similarly, this phenomenon is seen for the cases of $G = 2$ with various Bi_a values.

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Transient Temperature Analysis of Airplane Carbon Composite Disk Brakes

Ahmet Z. Şahin* and Ahmed Z. Al-Garni*
King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

Nomenclature

Bi = Biot number
h = convective heat transfer coefficient, W/m^2 K
k = thermal conductivity, W/mK
P = pressure, N/m^2
q'' = heat flux, W/m^2
R_i = inner radius of brake disk, m
R_o = outer radius of brake disk, m
T = temperature, K
t = time, s
z = axial coordinate, m
α = thermal diffusivity, m^2/s
δ = thickness of brake disk, m
ζ = dimensionless axial distance
θ = dimensionless temperature
μ = coefficient of friction
ρ = dimensionless radial distance
ρC = heat capacity, J/m^3 K
τ = dimensionless time
ω = angular speed, 1/s

Introduction

BECAUSE of outstanding high-temperature heat dissipating qualities, carbon–carbon advanced composites have become attractive materials for aircraft disk brake pads. The use of chemical vapor deposition (CVD) techniques in the preparation of composite structures has made considerable improvements possible in thermomechanical properties of carbon–carbon composite materials.[1] Carbon has a high heat ab-

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*Associate Professor, Mechanical Engineering Department.

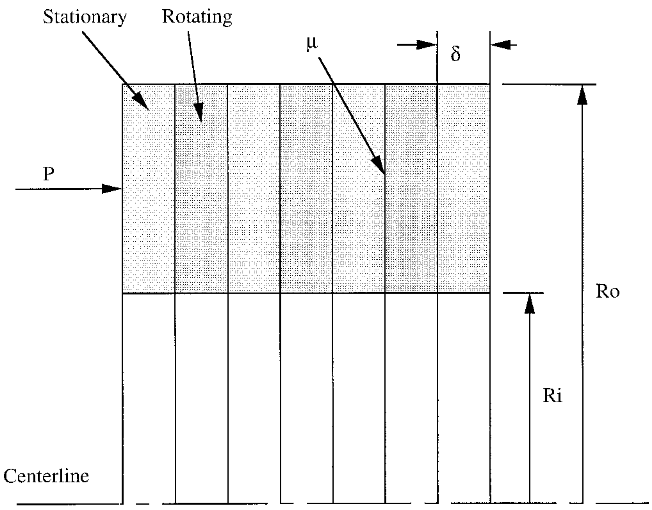

Fig. 1 Model for multiple disk brake assembly.

sorbing capability. Brake weight is reduced because of its high specific heat. When comparing the strengths of candidate brake materials (steel, beryllium, and carbon) as functions of temperature, carbon–carbon composites retain their strength at high temperatures while a considerable decrease in strength is observed in the other materials.[2]

A transient thermoelastic analysis of composite brake disks was made by Sonn et al.,[3] assuming constant thermophysical properties of the brake material. They showed that ρC is the main factor that influences the thermoelastic behavior, and the effects of thermal boundary conditions and convective heat transfer coefficient in the brake housing are negligible during the braking action because of small thermal diffusivity.

In an experimental study performed by Dowding et al.,[4] the dependence of thermal properties of carbon–carbon composite material with temperature was given. They found that their experimental results can be closely correlated in the form of quadratic relations within the temperature range of 30–600°C, with accuracies of 2 and 6% for thermal conductivity and heat capacity, respectively. In this temperature range, the heat capacity of carbon–carbon composite material showed an increase of more than 100%. Consequently, the thermal analysis of the carbon–carbon composite brake system must include variable thermal properties for accurate determination of thermal behavior.

In the present study, the transient thermal analysis of a multiple carbon–carbon composite aircraft brake system is given using variable thermal properties. The applied hydraulic pressure is assumed to be uniform on the contact areas but varying with time. The convection heat transfer coefficient is assumed to be constant on all external surfaces of the multiple disk assembly.

Analytical Model

Consider seven cylindrical brake disks of thickness δ and radii of R_i and R_o, respectively, as shown in Fig. 1. Rotating disks have an angular velocity ω, which is varying with time. At the surface of the intersection of the two disks, μ and P are assumed to be constant and uniform throughout the surface. The thermal heat capacity is given by ρC. The thermal conductivity and the heat transfer coefficient are given by k and h, respectively. The transversal symmetry is also assumed to be satisfied.

The governing equation for the thermal conduction in the brake disks is given by

$$\rho C \frac{\partial T_j}{\partial t} = \nabla(k\nabla T_j), \qquad j = 1, 2, \ldots, 7 \tag{1}$$

where T_j is the temperature of an individual disk (j = 1, 2, . . . , 7). The frictional heat generated on each of the six interfaces, $q'' = \mu P\omega r$, diffuses inside the brake disk material by conduction. All of the external surfaces are subjected to convective boundary conditions with h and ambient temperature T_∞.

The thermal conductivity and the volumetric heat capacity of the carbon–carbon composite material are given by[4]

$$\begin{aligned} k &= k_0(1 + 0.002366T - 0.2545 \times 10^{-5}T^2) \\ \rho C &= (\rho C)_0(1 + 0.00411T - 0.284 \times 10^{-5}T^2) \end{aligned} \tag{2}$$

respectively, where $k_0 = 3.195$, $(\rho C)_0 = 1.28 \times 10^6$, and T is in °C.

Applied braking pressure and angular velocity are assumed to vary linearly with time as

$$\frac{P}{P_0} = \begin{cases} 2.5t, & 0 \le t < 0.4s \\ 1, & 0.4 \le t < 1.0s \end{cases}$$

where $P_0 = 0.7 \times 10^6$ Pa and

$$\omega/\omega_0 = 1 - t, \qquad 0 < t < 1.0$$

where $\omega_0 = 500$ rad/s, respectively.

The solution of the preceding model is obtained by using a finite difference method. Although the convective heat transfer coefficient is assumed to be constant on all of the external surfaces of the assembly, variable heat transfer coefficient can easily be implemented in the numerical algorithm.

The following dimensionless parameters are used in writing the nodal equations in nondimensionalized form: $\rho = r/R_i$, $\zeta = z/R_i$, $\tau = \alpha_0 t/R_i^2$, and

$$\theta = \frac{T - T_\infty}{\mu P_0 \omega_0 R_i^2/k_0} \times 10^3$$

The constant factor 10^3 is used to decrease the round-off error in calculations.

The heat generation on the frictional surfaces is $q'' = \mu P\omega r$. This can be written in dimensionless form as

$$\frac{q''}{q''_0} = \left(\frac{P}{P_0}\right)\left(\frac{\omega}{\omega_0}\right)\rho$$

where $q''_0 = \mu P_0 \omega_0 R_i$.

Choosing equal grid spacing both in r and z directions ($\Delta z = \Delta r$), the nodal equation for a general inner node can be written as

$$\begin{aligned} \theta'_{i,j} = {} & \left[1 - 4\left(\frac{\alpha_{i,j}}{\alpha_0}\right)\frac{\Delta\tau}{\Delta\rho^2}\right]\theta_{i,j} + \left(\frac{\alpha_{i,j}}{\alpha_0}\right)\frac{\Delta\tau}{\Delta\rho^2}\Bigg(\theta_{i,j-1} + \theta_{i,j+1} \\ & + \left\{1 - \frac{\Delta\rho}{2[1 + (i-1)\Delta\rho]}\right\}\theta_{i-1,j} \\ & + \left\{1 + \frac{\Delta\rho}{2[1 + (i-1)\Delta\rho]}\right\}\theta_{i+1,j} + 10^3\left(\frac{q''_{i,j}}{q''_0}\right)\left(\frac{k_0}{k_{i,j}}\right)\Delta\rho\Bigg) \end{aligned} \tag{3}$$

where $\alpha_0 = k_0/(\rho C)_0$. It should be noted that the term $q''_{i,j}/q''_0$ is nonzero only on the nodes that are located on the frictional surfaces, and it is zero everywhere else in the domain. It can be shown that the time step to be selected to ensure stability is

$$\Delta\tau \le \frac{\Delta\rho^2}{4Bi\Delta\rho + 4 + \Delta\rho} \tag{4}$$

where $Bi = hR_i/k_0$.

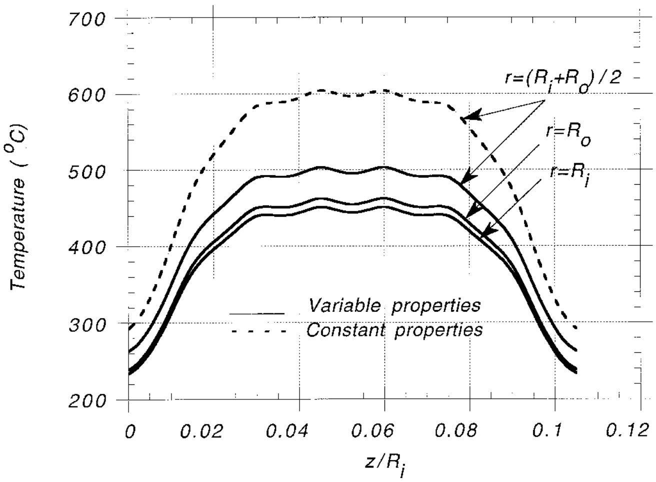

Fig. 2 Axial temperature profiles at fixed radial distances and time of 1 s.

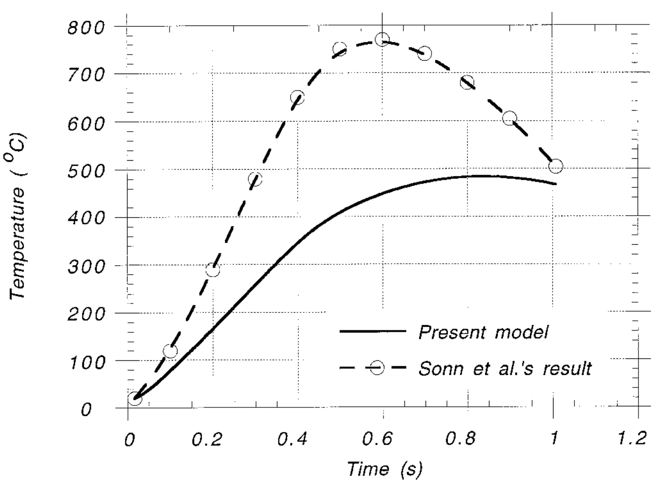

Fig. 3 Maximum temperature rise in the carbon–carbon composite disk brake assembly.

Discussion

Figure 2 shows the axial temperature profiles at three different radial distances and a fixed time of 1 s. Because of the cooling effect by convection, the inner and outer surfaces exhibit lower temperatures, while the maximum temperature profile lies somewhere between the midradius and the outer radius, because the frictional heat generation that is proportional to the radial distance increases toward the outer radius. The axial temperature distribution at midradius for the case of constant thermophysical properties is also included in Fig. 2 for comparison. When the thermophysical properties are evaluated at an average temperature of 300°C, a considerably high brake temperature variation is obtained. The difference in temperatures at 1 s can be as high as 20–30%. This indicates that variation of thermophysical properties with temperature is important and should be considered.

Figure 3 shows the temperature rise at the location where the temperature is maximum. A sharp increase in temperature is observed during the early time of braking action. The increase in temperature then slows down, and even a decrease in temperature can be obtained. This is the consequence of decreasing the angular speeds of the disks relative to one another and of the frictional heat generation associated with it. The history of the maximum temperature obtained using constant thermophysical properties as given by Sonn et al.[3] is also included in Fig. 3 for comparison.

The effect of h on the brake temperature variation has also been studied. It is found that the effect of h on the brake temperature is noticeable only in the vicinity of the outer surfaces where the brakes are exposed to convection. The internal areas of the disk brake assembly are not affected significantly with a change of h.

Conclusions

The transient temperature analysis of a multiple carbon–carbon aircraft disc brake system is obtained numerically using temperature-dependent thermophysical properties. It is shown that the specific heat is the main factor affecting the temperature distribution. An assumption of constant thermophysical properties can lead to 20–30% of inaccurate results. The maximum temperatures are obtained around the middle disk and at locations closer to the outer radius. The temperature rise is initially sharp and declines afterward. This is a result of time variations of angular velocity and pressure functions assumed in the analysis. The effect of convective heat transfer coefficient on the brake temperature variation is negligible.

Acknowledgment

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Exact Determination of Transient Cooling in Small Bodies by Nonlinear Natural Convection

Antonio Campo*

Idaho State University, Pocatello, Idaho 83209

and

Abraham Salazar†

University of Kentucky, Lexington, Kentucky 40506

Nomenclature

A	= surface area
Bi	= Biot number, $\bar{h}L_c/k_s$
c	= specific heat capacity
g	= gravitational acceleration
$\bar{h}$	= space–mean convection coefficient
K_{sf}	= solid–fluid thermal conductivity ratio, k_s/k_f
k	= thermal conductivity

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*Professor, Nuclear Engineering Department. Member AIAA.

†Graduate Student, Mechanical Engineering Department.